

SO(9,1) invariant matrix formulation of supermembrane

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Abstract

An $SO(9,1)$ invariant formulation of an 11-dimensional supermembrane is presented by combining an $SO(10,1)$ invariant treatment of reparametrization symmetry with an $SO(9,1)$ invariant $\theta_R = 0$ gauge of κ -symmetry. The Lagrangian thus defined consists of polynomials in dynamical variables (up to quartic terms in X^μ and up to the eighth power in θ), and reparametrization BRST symmetry is manifest. The area preserving diffeomorphism is consistently incorporated and the area preserving gauge symmetry is made explicit. The $SO(9,1)$ invariant theory contains terms which cannot be induced by a naive dimensional reduction of higher dimensional supersymmetric Yang-Mills theory. The $SO(9,1)$ invariant Hamiltonian and the generator of area preserving diffeomorphism together with the supercharge are matrix regularized by applying the standard procedure. As an application of the present formulation, we evaluate the possible central charges in superalgebra both in path integral and in canonical (Dirac) formalism, and we find only the two-form charge $[X^\mu, X^\nu]$.

1 Introduction

A matrix formulation [1, 2] of an 11-dimensional supermembrane [3]-[7] received much attention recently in connection with a possible non-perturbative analysis of the so-called M-theory [8, 9, 10]. The matrix formulation so far is based on the light-cone gauge formulation [11], which simplifies much the structure of the action. Recently, we presented a Lorentz covariant matrix formulation of a bosonic membrane[12], by extending a full covariant BRST formulation of the bosonic membrane in Ref.[13]. In the present paper, we present an $SO(9, 1)$ invariant formulation of the supermembrane by combining the manifestly Lorentz covariant treatment of reparametrization symmetry with the $SO(9, 1)$ invariant $\theta_R = 0$ gauge of κ -symmetry, which has been proposed recently[14, 15, 16]. The theory thus formulated consists of finite polynomials in dynamical variables, and reparametrization BRST symmetry is explicit. The area preserving diffeomorphism and the area preserving gauge symmetry are also made manifest. The matrix formulation is obtained by applying the standard procedure. This formulation, which preserves most of the Lorentz boost symmetry, inherits much of the structure of the original supermembrane, and for example, it contains terms which cannot be obtained by a naive dimensional reduction of higher dimensional supersymmetric Yang-Mills theory. Another characteristic of the present formulation is that the $\theta_R = 0$ gauge becomes singular for a naive “double dimensional reduction” of the supermembrane to the Type IIA string; if such a reduction exists, it should be non-perturbative one in the present formulation. As an application of this formulation, we examine the possible central charges in superalgebra in path integral as well as in canonical (Dirac) formulations. We find only the (possible) two-form central charges but no five-form charges, as is expected for a supermembrane. We emphasize that the light-cone formulation [1] and the present $SO(9, 1)$ invariant formulation, though Lagrangians have quite different appearance, in fact describe an identical theory in the domain where both of the gauge conditions are well-defined.

To make this paper self-contained, we here recapitulate the basic definition of the supermembrane[3][7]: The action consists of two terms, the Dirac term and the Wess-Zumino term. The Dirac term S_D is written as

$$\begin{aligned} S_D &= \int_W d^3\sigma \frac{1}{2} \sqrt{-g} (1 - g^{ab} h_{ab}) \\ &= \int_W d^3\sigma \frac{1}{2} (-\det \tilde{g} - \tilde{g}^{ab} h_{ab}) \end{aligned} \quad (1.1)$$

where $\tilde{g}^{ab} = \sqrt{-g} g^{ab}$, and σ^a ($a = 0, 1, 2$) are membrane world-volume coordinates. The

variables h_{ab} are induced metric on the membrane world-volume W

$$h_{ab} = \eta_{\mu\nu} \Pi_a^\mu \Pi_b^\nu \quad (1.2)$$

where the flat $D = 11$ target space-time metric is defined by $\eta_{\mu\nu} = \text{diag}(-1, +1, \dots, +1)$, and

$$\Pi_a^\mu = \partial_a X^\mu - i\bar{\theta}\Gamma^\mu\partial_a\theta \quad (1.3)$$

with a 32-component Majorana spinor θ and $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$.

The Wess-Zumino term is written as

$$S_{WZ} = - \int_W \frac{1}{2} a_3 \quad (1.4)$$

with a 3-form a_3

$$a_3 = \Sigma_{\mu\nu} \alpha_2^{\mu\nu} = \Sigma_{\mu\nu} \left[\Pi^\mu \Pi^\nu - \Sigma^\mu \left(\Pi^\nu - \frac{1}{3} \Sigma^\nu \right) \right] \quad (1.5)$$

The 3-form a_3 is a potential of a closed 4-form h_4

$$h_4 = da_3 = d\Sigma_{\mu\nu} \Pi^\mu \Pi^\nu = -i d\bar{\theta} \Gamma_{\mu\nu} d\theta \Pi^\mu \Pi^\nu \quad (1.6)$$

In this paper we often use the form notation, which simplifies many of the equations. We here defined the basic 1-forms by

$$\Sigma_{\mu.. \nu} = i d\bar{\theta} \Gamma_{\mu.. \nu} \theta \quad (1.7)$$

with the statistics convention $d\theta = \partial_a \theta d\sigma^a = d\sigma^a \partial_a \theta$, and

$$\Pi^\mu = \Pi_a^\mu d\sigma^a = dX^\mu + \Sigma^\mu \quad (1.8)$$

Our definition of exterior derivative is the standard one on the *bosonic* manifold, i.e.,

$$d(A_p \wedge B_q) = dA_p \wedge B_q + (-1)^p A_p \wedge dB_q \quad (1.9)$$

for p -form A_p and q -form B_q .

The supermembrane action has several symmetries : reparametrization symmetry, which is manifest in the action, and global SUSY and local κ -symmetries. The target space global SUSY is defined by

$$\begin{aligned} \delta_{SUSY} X^\mu &= i\bar{\epsilon}\Gamma^\mu\theta \equiv l^\mu \\ \delta_{SUSY} \theta &= \epsilon \end{aligned} \quad (1.10)$$

and Π^μ is invariant, $\delta_{SUSY}\Pi^\mu = 0$. The κ -symmetry is defined by

$$\begin{aligned}\delta_\kappa\theta &= (1+\gamma)\kappa \\ \delta_\kappa X^\mu &= i\bar{\theta}\Gamma^\mu\delta_\kappa\theta \\ \delta_\kappa\tilde{g}^{ab} &= -K_c^a\tilde{g}^{cb}\end{aligned}\tag{1.11}$$

where γ , which is a world-volume analogue of γ_5 , is defined by

$$\gamma = \frac{1}{3!}\epsilon^{abc}\gamma_a\gamma_b\gamma_c\tag{1.12}$$

in terms of the induced γ -matrices

$$\gamma_a = \Pi_a^\mu\Gamma_\mu \quad , \quad \{\gamma_a, \gamma_b\} = 2h_{ab}\tag{1.13}$$

The convention of the anti-symmetric tensor is

$$\varepsilon_{012} = 1 \quad , \quad \varepsilon^{012} = -1\tag{1.14}$$

and

$$\epsilon_{abc} = \sqrt{-g}\varepsilon_{abc} \quad , \quad \epsilon^{abc} = \frac{1}{\sqrt{-g}}\varepsilon^{abc}\tag{1.15}$$

The coefficient K in (1.11) is given by

$$\begin{aligned}K_b^a &= i\epsilon^{acd}\partial_b\bar{\theta}\gamma_{cd}\delta_\kappa\theta + (a \leftrightarrow b) \\ &\quad - \frac{2i}{3}(\partial_e\bar{\theta}\gamma^e\kappa)[f(1,1) + f(1,\bar{h}) + f(\bar{h},\bar{h})]_b^a\end{aligned}\tag{1.16}$$

where

$$f(A,B) = \text{tr}A\text{tr}B - \text{tr}AB + AB + BA - A\text{tr}B - B\text{tr}A\tag{1.17}$$

and $\bar{h}_b^a = g^{ac}h_{cb}$. We raise and lower the world-volume indices a, b by the metric g_{ab} . We note that

$$\gamma^2 = \det \bar{h} = \frac{1}{3!}\text{tr}[\bar{h}f(\bar{h},\bar{h})]\tag{1.18}$$

before the use of equations of motion for g_{ab} .

2 $SO(9,1)$ invariant gauge fixing

In this section we define an $SO(9,1)$ invariant gauge for the supermembrane. The 32-component Majorana spinor is an irreducible representation of $SO(10,1)$, and any algebraic gauge fixing of κ -symmetry generally breaks the full $SO(10,1)$ symmetry. The basic

idea of $\theta_R \equiv \frac{1}{2}(1 - \Gamma_{11})\theta = 0$ gauge is to decompose

$$\begin{aligned}\theta &= \theta_L + \theta_R \\ \mathbf{32} &= \mathbf{16}_L \oplus \mathbf{16}_R \\ SO(10, 1) &\supset SO(9, 1)\end{aligned}\tag{2.1}$$

We also decompose $\{X^\mu\}$ as $X^\mu = (X^m, X^{11})$, where we use μ for a 11-dimensional index and m for a 10-dimensional index. Note that the eleventh element Γ_{11} of $D = 11$ Γ -matrices $\{\Gamma^\mu\}$ is identified with $D = 10$ chirality matrix $\Gamma_0 \cdots \Gamma_9$. The $\theta_R = 0$ gauge fixing of κ -symmetry [14, 15, 16] is not $D = 11$ covariant but it is $D = 10$ covariant. This gauge preserves most of the Lorentz boost symmetry compared to the light-cone gauge, which is based on $SO(10, 1) \supset SO(1, 1) \times SO(9)$.

In the following, we put $\hat{}$ on the objects on the world-volume which consist of 10-dimensional variables. For example,

$$\begin{aligned}\hat{\gamma}_a &= \Pi_a^m \Gamma_m \\ \hat{h}_{ab} &= \eta_{mn} \Pi_a^m \Pi_b^n\end{aligned}\tag{2.2}$$

The matrix γ in (1.12) is written in a $D = 10$ notation as

$$\begin{aligned}\gamma &= \hat{\gamma} + \frac{1}{2} \Gamma_{11} \Pi_a^{11} \epsilon^{abc} \hat{\gamma}_{bc} \\ &= (1 - \hat{\rho} \Gamma_{11}) \hat{\gamma}\end{aligned}\tag{2.3}$$

with

$$\hat{\rho} = \Pi_a^{11} (\hat{h}^{-1})^{ab} \hat{\gamma}_b\tag{2.4}$$

We used the relation $\hat{\gamma}_a \hat{\gamma} = \frac{1}{2} \hat{h}_{ab} \epsilon^{bcd} \hat{\gamma}_{cd}$. The matrix γ contains terms with both even and odd Γ^m 's, and it has no definite chirality-flip property in 10-dimensions. The “irreducible” κ -symmetry is to choose $\kappa^{ir} = \hat{\gamma} \kappa_R / \hat{\gamma}^2$ and

$$\begin{aligned}\delta_\kappa^{ir} \theta &= (1 + \gamma) \kappa^{ir} \\ &= (1 + \gamma_{ir}) \kappa_R\end{aligned}\tag{2.5}$$

where

$$\gamma_{ir} = \hat{\rho} + \frac{\hat{\gamma}}{\hat{\gamma}^2}\tag{2.6}$$

δ_κ^{ir} is essentially the κ -symmetry of a $D2$ -brane [17]. In the chiral notation, $\Gamma_{11} \theta_{L,R} = \pm \theta_{L,R}$, we have

$$\delta_\kappa^{ir} \theta_L = \gamma_{ir} \kappa_R\tag{2.7}$$

$$\delta_\kappa^{ir} \theta_R = \kappa_R\tag{2.8}$$

The matrix γ_{ir} contains odd Γ_m 's only, and consequently, γ_{ir} flips chirality in a 10-dimensional sense. The relation $\delta_{\kappa}^{ir}\theta_R = \kappa_R$ shows that the variable θ_R is identified with the gauge parameter κ_R itself, which forms the basis of the $\theta_R = 0$ gauge [14, 15, 16]. Note that $\hat{\gamma} = 0$ for a naive “double dimensional reduction”, and the gauge $\theta_R = 0$ becomes singular in such a limit.

By extending the covariant gauge fixing of the bosonic membrane, we adopt the gauge condition

$$\tilde{g}^{0a} + \delta^{0a} = 0, \quad \theta_R = 0 \quad (2.9)$$

The first condition corresponds to an orthogonal decomposition of the 3-dimensional membrane world-volume W into $W = \mathbf{R} \times \Sigma$, where \mathbf{R} and Σ are time and space part of the membrane world-volume, respectively. We use $\tau \equiv \sigma^0$ and σ^k ($k = 1, 2$) for coordinates on $\mathbf{R} \times \Sigma$. The gauge fixing and Faddeev-Popov terms are

$$\begin{aligned} \mathcal{L}_g &= \delta_{BRST} [b_a(\tilde{g}^{0a} + \delta^{0a}) + \bar{\beta}_L \theta_R] \\ &= N_a(\tilde{g}^{0a} + \delta^{0a}) + \bar{\xi}_L \theta_R \\ &\quad + i b_a [\partial_b (c^b \tilde{g}^{0a}) - \tilde{g}^{ba} \partial_b c^0 - \tilde{g}^{0b} \partial_b c^a] \end{aligned} \quad (2.10)$$

with

$$\bar{\xi}_L \equiv \bar{\xi}_L + i \partial_a (\bar{\beta}_L c^a) \quad (2.11)$$

One may understand (2.10) as formally obtained from

$$\begin{aligned} \mathcal{L}_g &= N_a(\tilde{g}^{0a} + \delta^{0a}) + \bar{\xi}_L \theta_R \\ &\quad + i b_a [\partial_b (c^b \tilde{g}^{0a}) - \tilde{g}^{ba} \partial_b c^0 - \tilde{g}^{0b} \partial_b c^a - K_b^a (\kappa = \hat{\gamma} \gamma_R / \hat{\gamma}^2) \tilde{g}^{0b}] \\ &\quad + \bar{\beta}_L (\gamma_R - i c^a \partial_a \theta_R) \\ &= N_a(\tilde{g}^{0a} + \delta^{0a}) + \bar{\xi}_L \theta_R \\ &\quad + i b_a [\partial_b (c^b \tilde{g}^{0a}) - \tilde{g}^{ba} \partial_b c^0 - \tilde{g}^{0b} \partial_b c^a - K_b^a (\kappa = \hat{\gamma} \gamma_R / \hat{\gamma}^2) \tilde{g}^{0b}] \\ &\quad + \bar{\beta}_L \gamma_R \end{aligned} \quad (2.12)$$

after partial integration and then path integrating out the *non-propagating* ghost sector $\bar{\beta}_L \gamma_R$; this procedure is analogous to that of the unitary gauge in the Higgs mechanism. The variable ξ_L is a Lagrangian multiplier to impose $\theta_R = 0$, and γ_R is a Faddeev-Popov ghost for κ -symmetry. By this way, only the following reparametrization BRST symmetry remains in (2.10)

$$\delta X^\mu = -i \epsilon c^a \partial_a X^\mu, \quad \delta \theta = -i \epsilon c^a \partial_a \theta$$

$$\begin{aligned}
\delta c^a &= -i\epsilon c^b \partial_b c^a \quad , \quad \delta b_a = \epsilon N_a \\
\delta N_a &= 0 \quad , \quad \delta \tilde{g}^{ab} = -i\epsilon [\partial_c (c^c \tilde{g}^{ab}) - \tilde{g}^{cb} \partial_c c^a - \tilde{g}^{ac} \partial_c c^b] \\
\delta \bar{\tilde{\xi}}_L &= -i\epsilon \partial_a (c^a \bar{\tilde{\xi}}_L),
\end{aligned} \tag{2.13}$$

where the transformation law of $\bar{\tilde{\xi}}_L$ is induced by the original transformation law of the Nakanishi-Lautrup multiplet

$$\delta \bar{\beta}_L = \epsilon \bar{\xi}_L \quad , \quad \delta \bar{\xi}_L = 0 \tag{2.14}$$

In some of the analysis of BRST symmetry[18], it is convenient to revive an extra (redundant) variable β_L by re-writing

$$\bar{\tilde{\xi}}_L \theta_R \rightarrow \bar{\xi}_L \theta_R - i \bar{\beta}_L c^a \partial_a \theta_R \tag{2.15}$$

In passing, we note that the choice of the gauge $\tilde{g}^{00} = -\rho(\sigma^1, \sigma^2)$ [12] instead of $\tilde{g}^{00} = -1$ in (2.9) introduces a density ρ on Σ described in [1]. In this paper, we work with the gauge choice (2.9).

3 Gauge fixed action

If we integrate out \tilde{g}^{ab} and N_a in the total action with the above gauge fixing Lagrangian (2.10), we obtain the Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{WZ} + \mathcal{L}_{gh} \tag{3.1}$$

where

$$\begin{aligned}
\mathcal{L}_0 &= \frac{1}{2}(\Pi_0^\mu)^2 - \frac{1}{2} \det G_{kl} \\
\mathcal{L}_{gh} &= ib_0(\partial_0 c^0 - \text{div} \mathbf{c}) + i(\mathbf{b}, \partial_0 \mathbf{c}) + \bar{\tilde{\xi}}_L \theta_R
\end{aligned} \tag{3.2}$$

with

$$G_{kl} = \Pi_k^\mu \Pi_{\mu l} + ib_k \partial_l c^0 + ib_l \partial_k c^0 \tag{3.3}$$

We defined the 1-form \mathbf{b} and a vector field \mathbf{c} on Σ by

$$\mathbf{b} = b_k d\sigma^k \quad , \quad \mathbf{c} = c^k \partial_k \tag{3.4}$$

and $(\mathbf{b}, \partial_0 \mathbf{c})$ stands for an inner product. The Wess-Zumino term \mathcal{L}_{WZ} , which is independent of the metric, is not influenced in this procedure

$$\begin{aligned}
\mathcal{L}_{WZ} &= \Pi_0^\mu \{ \Sigma_{\mu\nu}, dX^\nu + \frac{1}{2} \Sigma^\nu \} \\
&\quad - \frac{1}{2} \Sigma_0^{\mu\nu} \left(\{ X_\mu, X_\nu \} + \{ dX_\mu + \frac{1}{3} \Sigma_\mu, \Sigma_\nu \} \right) \\
&\quad - \frac{1}{2} \Sigma_0^\mu \{ \Sigma_{\mu\nu}, dX^\nu + \frac{1}{3} \Sigma^\nu \}
\end{aligned} \tag{3.5}$$

We here defined “Poisson bracket” of two functions on Σ by

$$\{f, g\} = \varepsilon^{kl} \partial_k f \partial_l g = \partial_1 f \partial_2 g - \partial_2 f \partial_1 g \quad (3.6)$$

where one regards the variables (σ^1, σ^2) on Σ as canonically conjugate variables. In this paper, we use the bracket $\{f, g\}$ to denote the “Poisson bracket” in this sense; for the conventional *true* Poisson bracket, we attach a suffix such as $\{p, x\}_P$ to the bracket.

We also defined the “Poisson bracket” of 1-forms A and B on Σ by

$$\{A, B\} \equiv *(A \wedge B) = \varepsilon^{kl} A_k B_l = -\{B, A\} \quad (3.7)$$

For two exact 1-forms, the “Poisson bracket of 1-forms” becomes the ordinary Poisson bracket of functions

$$\{df, dg\} = \{f, g\} \quad (3.8)$$

For a general 1-form and an exact 1-form, we have

$$\{A, df\} = -(A, \vec{f}) \quad (3.9)$$

where $\vec{f} = \partial_k f \varepsilon^{kl} \partial_l = \partial_1 f \partial_2 - \partial_2 f \partial_1$ is a Hamiltonian vector field associated with f .

The reparametrization BRST symmetry now becomes

$$\begin{aligned} \delta X^\mu &= -i\epsilon c^a \partial_a X^\mu, & \delta \theta &= -i\epsilon c^a \partial_a \theta \\ \delta c^a &= -i\epsilon c^b \partial_b c^a, & \delta b_a &= \epsilon B_a \\ \delta \tilde{\xi}_L &= -i\epsilon \partial_a (c^a \tilde{\xi}_L), \end{aligned} \quad (3.10)$$

where

$$B_0 = \frac{1}{2} \Pi_0^\mu \Pi_{\mu 0} + \frac{1}{2} \det G_{kl} + 2ib_0 \partial_0 c^0 + i\partial_a b_0 c^a + ib_k \partial_0 c^k \quad (3.11)$$

$$B_k = \Pi_0^\mu \Pi_{\mu k} + i\partial_a b_k c^a + ib_k \partial_0 c^0 + ib_l \partial_k c^l \quad (3.12)$$

The variables B_a , up to equations of motion, correspond to the energy- momentum tensor T_{0a} on the world volume. The BRST charge is given by

$$\begin{aligned} Q_{BRST} &= \int_\Sigma d^2\sigma \left[c^0 \left(\frac{1}{2} \Pi_0^\mu \Pi_{\mu 0} + \frac{1}{2} \det G_{kl} \right) + c^k \Pi_0^\mu \Pi_{\mu k} \right. \\ &\quad \left. - ib_0 (c^0 \partial_k c^k + c^k \partial_k c^0) - ib_k c^l \partial_l c^k - c^0 \tilde{\xi}_L \theta_R \right] \end{aligned} \quad (3.13)$$

The above Lagrangian is regarded as a supersymmetrization of the bosonic Lagrangian in Ref.[13].

The factor $\det G_{kl}$ can be written as

$$\det G_{kl} = \frac{1}{2} \{ \Pi^\mu, \Pi^\nu \} \{ \Pi_\mu, \Pi_\nu \} + 2i \{ \Pi^\mu, \mathbf{b} \} \{ \Pi_\mu, dc^0 \} - 3(\mathbf{b}, \vec{c}^0)^2 \quad (3.14)$$

All the Poisson brackets of 1-forms, which appear in (3.5) and (3.14), are reduced to the Poisson brackets of functions, for example,

$$\{\Sigma^\mu, \Sigma^\nu\} = \bar{\theta}\Gamma^\mu\{\theta, \bar{\theta}\}\Gamma^\nu\theta = \theta^\alpha\Gamma_{\alpha\beta}^\mu\{\theta^\beta, \theta^\gamma\}\Gamma_{\gamma\delta}^\nu\theta^\delta \quad (3.15)$$

$$\{dX^\mu, \Sigma^\nu\} = i\{X^\mu, \bar{\theta}\}\Gamma^\nu\theta \quad (3.16)$$

We have thus established an important fact: All the terms in the above Lagrangian, which contain derivatives with respect to σ^k , except for some of the ghost terms are written in terms of the Poisson bracket of functions.

If we further integrate over $\tilde{\xi}_L$ and thus fix the gauge $\theta_R = 0$ strictly, the Lagrangian is further simplified to

$$\mathcal{L}' = \mathcal{L}'_0 + \mathcal{L}'_{WZ} + \mathcal{L}'_{gh} \quad (3.17)$$

where

$$\begin{aligned} \mathcal{L}'_0 &= \frac{1}{2}(\Pi_0^m)^2 + \frac{1}{2}(\partial_0 X^{11})^2 - \frac{1}{2}\det G_{kl} \\ \mathcal{L}'_{WZ} &= -\Sigma_m dX^m dX^{11} \\ &= i\bar{\theta}_L \Gamma_m [\partial_0 \theta_L \{X^m, X^{11}\} + \partial_0 X^m \{X^{11}, \theta_L\} + \partial_0 X^{11} \{\theta_L, X^m\}] \\ \mathcal{L}'_{gh} &= ib_0(\partial_0 c^0 - \text{div} \mathbf{c}) + i(\mathbf{b}, \partial_0 \mathbf{c}) \end{aligned} \quad (3.18)$$

with

$$\Pi^m = dX^m + \Sigma^m = dX^m - i\bar{\theta}_L \Gamma^m d\theta_L \quad (3.19)$$

$$G_{kl} = \Pi_k^m \Pi_{ml} + \partial_k X^{11} \partial_l X^{11} + ib_k \partial_l c^0 + ib_l \partial_k c^0 \quad (3.20)$$

The Wess-Zumino term \mathcal{L}'_{WZ} is rewritten as

$$\mathcal{L}'_{WZ} = i\Pi_0^m \bar{\theta}_L \Gamma_m \{X^{11}, \theta_L\} + i\partial_0 X^{11} \bar{\theta}_L \Gamma_m \{\theta_L, X^m\} - i\bar{\theta}_L \Gamma^m \partial_0 \theta_L Y_m \quad (3.21)$$

with

$$Y_m = -\{\Pi_m, dX^{11}\} = -\{X_m, X^{11}\} + i\bar{\theta}_L \Gamma_m \{\theta_L, X^{11}\} \quad (3.22)$$

The factor $\det G_{kl}$ in (3.18) is expanded as

$$\begin{aligned} \det G_{kl} &= \frac{1}{2}\{\Pi^m, \Pi^n\}^2 + \{\Pi^m, dX^{11}\}^2 \\ &\quad + 2i\{\Pi^m, \mathbf{b}\}\{\Pi_m, dc^0\} + 2i(\mathbf{b}, X^{\vec{11}})\{X^{11}, c^0\} - 3(\mathbf{b}, c^{\vec{0}})^2 \end{aligned} \quad (3.23)$$

The BRST symmetry is finally reduced to

$$\begin{aligned} \delta X^\mu &= -i\epsilon c^a \partial_a X^\mu \quad , \quad \delta \theta_L = -i\epsilon c^a \partial_a \theta_L \\ \delta c^a &= -i\epsilon c^b \partial_b c^a \quad , \quad \delta b_a = \epsilon B_a \end{aligned} \quad (3.24)$$

where

$$B_0 = \frac{1}{2} \left[(\Pi_0^m)^2 + (\partial_0 X^{11})^2 + \det G_{kl} \right] + 2ib_0 \partial_0 c^0 + i\partial_a b_0 c^a + ib_k \partial_0 c^k \quad (3.25)$$

$$\mathbf{B} = \Pi_0^m \Pi_m + \partial_0 X^{11} dX^{11} + id(b_0 c^0) + ib_0 dc^0 + i\mathbf{b}c - i\mathcal{L}_c \mathbf{b} \quad (3.26)$$

with a Lie derivative of a 1-form $(\mathcal{L}_c \mathbf{b})_k = c^l \partial_l b_k + \partial_k c^l b_l$ and $\mathbf{B} = B_k d\sigma^k$. We also used the equations of motion, $\partial_0 c^0 = \text{div} \mathbf{c}$ and $\partial_0 b_k = \partial_k b_0$, and we defined a new variable

$$c \equiv \text{div} \mathbf{c} = \partial_k c^k \quad (3.27)$$

The BRST charge is then written as

$$\begin{aligned} Q_{BRST} = \int_{\Sigma} d^2\sigma \left[\frac{1}{2} c^0 (\Pi_0^m \Pi_{m0} + (\partial_0 X^{11})^2 + \det G_{kl}) \right. \\ \left. + c^k (\Pi_0^m \Pi_{mk} + \partial_0 X^{11} \partial_k X^{11}) - ib_0 (c^0 \partial_k c^k + c^k \partial_k c^0) - ib_k c^l \partial_l c^k \right] \end{aligned} \quad (3.28)$$

Lagrangians (3.1) and (3.17) are physically equivalent; Lagrangian (3.1) exhibits more symmetry, whereas Lagrangian (3.17) contains the minimum set of variables in the present $\theta_R = 0$ gauge.

4 Superalgebra in path integral formulation

We now examine the supersymmetry algebra in our formulation. We start with the path integral analysis. In this section, we use the action *before* integrating out N_a and \tilde{g}^{ab} :

$$S = S_D + S_{WZ} + \int_W d^3\sigma \mathcal{L}_g \quad (4.1)$$

with the gauge fixing Lagrangian \mathcal{L}_g in (2.10). The variation of the action under a localized SUSY is [1, 7]

$$\delta_\epsilon S = \int_W d^3\sigma \left[i\partial_a \bar{\epsilon} J^a + \bar{\epsilon} \tilde{\xi}_L \right] + \int_W \frac{1}{2} d\beta_2 \quad (4.2)$$

where,

$$\begin{aligned} J^a = -2\tilde{g}^{ab} \Pi_b^\mu \Gamma_\mu \theta \\ + \varepsilon^{abc} \left[\Gamma_{\mu\nu} \theta \Pi_b^\mu \Pi_c^\nu + \frac{4i}{3} (\Gamma^\mu \theta \bar{\theta} \Gamma_{\mu\nu} \partial_b \theta + \Gamma_{\mu\nu} \theta \bar{\theta} \Gamma^\mu \partial_b \theta) (\Pi_c^\nu - \frac{2}{5} \Sigma_c^\nu) \right] \end{aligned} \quad (4.3)$$

and

$$\beta_2 = l_{\mu\nu} \left[\alpha_2^{\mu\nu} + 2\Sigma^\mu \left(-\frac{1}{3} \Pi^\nu + \frac{1}{5} \Sigma^\nu \right) \right] + l^\mu \left[\frac{1}{3} \Sigma_{\mu\nu} \left(\Pi^\nu - \frac{4}{5} \Sigma^\nu \right) \right] \quad (4.4)$$

with $l^{\mu\nu} \equiv i\epsilon\Gamma^{\mu\nu}\theta$. In the evaluation of (4.3) and (4.4), we used the identity

$$\Gamma_{(\alpha\beta}^{\mu\nu}\Gamma_{\mu\gamma\delta)} = 0 \quad (4.5)$$

where we symmetrize with respect to all the four spinor indices. The relation $\bar{\xi}\Gamma_\mu\eta = -\bar{\eta}\Gamma_\mu\xi$ for two Majorana spinors is also often used. The term $d\beta_2$ may be dropped for a *closed* membrane, and we consider only this case in the following. The equation of motion for the supercurrent is obtained from (4.2) as

$$\partial_a J^a = -i\tilde{\xi}_L \quad (4.6)$$

Physical information can be extracted from the equal-time commutator of these (broken-symmetry) supercharges in the path integral framework by using the Bjorken-Johnson-Low (BJL) prescription [18].

A SUSY transform of the supercurrent for a τ -dependent but σ^k -independent parameter $\epsilon(\tau)$ is given by

$$\begin{aligned} -\delta_\epsilon(\bar{\eta}J^0) &= 4\tilde{g}^{00}\partial_0\bar{\epsilon}\Gamma^\mu\theta\tilde{l}_\mu + 2\bar{\epsilon}\Gamma^\mu\eta P_\mu - \bar{\epsilon}\Gamma^{\mu\nu}\eta\{X_\mu, X_\nu\} \\ &\quad - *d\left[i(l_\mu\tilde{l}^{\mu\nu} + l^{\mu\nu}\tilde{l}_\mu)\left(\frac{2}{3}dX_\nu + \frac{1}{5}\Sigma_\nu\right) + \frac{2i}{15}(\tilde{l}^{\mu\nu}\Sigma_\mu + \tilde{l}_\mu\Sigma^{\mu\nu})l_\nu\right] \end{aligned} \quad (4.7)$$

where the exterior derivative and the wedge operation are defined on the 2-dimensional space Σ of the membrane world-volume. We defined $l^\mu = i\bar{\epsilon}\Gamma^\mu\theta$, $\tilde{l}^\mu = i\bar{\eta}\Gamma^\mu\theta$ and the momentum density

$$P^\mu = \frac{\delta\mathcal{L}}{\delta(\partial_0 X_\mu)} = -\tilde{g}^{0a}\Pi_a^\mu + \{\Sigma^{\mu\nu}, dX_\nu + \frac{1}{2}\Sigma_\nu\} \quad (4.8)$$

The third term in (4.7) represents a (possible) central charge of the supermembrane. The fourth term in (4.7) is a total derivative, and one may think of it as a central charge also. However the fourth term consists of terms containing θ , and it may not give a non-vanishing contribution at the boundary. For this reason we tentatively drop it in the following.

The Ward-Takahashi identity for SUSY generators is given by

$$\begin{aligned} &\frac{\partial}{\partial\tau}\langle T^*\bar{\epsilon}Q(\tau)\bar{\eta}Q(\tilde{\tau})\rangle \\ &= 4i\frac{\partial}{\partial\tau}\delta(\tau - \tilde{\tau})\left\langle\int_\Sigma d^2\tilde{\sigma}\tilde{g}^{00}l^\mu\tilde{l}_\mu(\tilde{\tau}, \tilde{\sigma}^k)\right\rangle \\ &\quad + \delta(\tau - \tilde{\tau})\left\langle\int_\Sigma d^2\tilde{\sigma}\left[2\bar{\epsilon}\Gamma^\mu\eta P_\mu(\tilde{\tau}, \tilde{\sigma}^k) - \bar{\epsilon}\Gamma^{\mu\nu}\eta\{X_\mu, X_\nu\}(\tilde{\tau}, \tilde{\sigma}^k)\right]\right\rangle \\ &\quad + \left\langle -i\int_\Sigma d^2\sigma T^*\bar{\epsilon}\xi_L(\tau, \sigma^k)\bar{\eta}Q(\tilde{\tau})\right\rangle \end{aligned} \quad (4.9)$$

In path integral this relation is obtained by starting with the expression

$$\langle \bar{\eta} Q(\tilde{\tau}) \rangle \quad (4.10)$$

and applying the localized SUSY variation as in (4.7) and also the corresponding variation of the action. The above identity is written in terms of the T^* -product, and we can identify the relation in terms of the T -product by using the B JL prescription. This procedure corresponds to removing the terms with $\partial_\tau \delta(\tau - \tilde{\tau})$ [18], and we obtain

$$\begin{aligned} & \frac{\partial}{\partial \tau} \langle T \bar{\epsilon} Q(\tau) \bar{\eta} Q(\tilde{\tau}) \rangle \\ &= \delta(\tau - \tilde{\tau}) \left\langle \int_{\Sigma} d^2 \tilde{\sigma} \left[2 \bar{\epsilon} \Gamma^\mu \eta P_\mu(\tilde{\tau}, \tilde{\sigma}^k) - \bar{\epsilon} \Gamma^{\mu\nu} \eta \{X_\mu, X_\nu\}(\tilde{\tau}, \tilde{\sigma}^k) \right] \right\rangle \\ &+ \left\langle -i \int_{\Sigma} d^2 \sigma T \bar{\epsilon} \xi_L(\tau, \sigma^k) \bar{\eta} Q(\tilde{\tau}) \right\rangle \end{aligned} \quad (4.11)$$

By explicitly operating the time derivative operation in this relation and using the equation of motion (4.6), the *equal time* commutator of supercharge is obtained as

$$[\bar{\epsilon} Q(\tau), \bar{\eta} Q(\tau)] = \int_{\Sigma} d^2 \sigma \bar{\epsilon} \left(2 \Gamma^\mu P_\mu(\tau, \sigma^k) - \Gamma^{\mu\nu} \{X_\mu, X_\nu\}(\tau, \sigma^k) \right) \eta \quad (4.12)$$

The gauge fixing term, which breaks supersymmetry, does not influence this equal-time SUSY algebra. This is analogous to the old chiral $SU(3) \times SU(3)$ algebra where the soft breaking of symmetry by a mass term does not influence the chiral charge algebra. We here note that the derivation of (4.12) goes through without modification for the Lagrangian (3.1) also.

If we compare the above algebra with the most general $D = 11$ super algebra

$$\{Q_\alpha, Q_\beta\} = 2 \left[P_\mu \Gamma_{\alpha\beta}^\mu + \frac{1}{2} Z_{\mu_1 \mu_2} \Gamma_{\alpha\beta}^{\mu_1 \mu_2} + \frac{1}{5!} Z_{\mu_1 \dots \mu_5} \Gamma_{\alpha\beta}^{\mu_1 \dots \mu_5} \right] \quad (4.13)$$

we can identify the central charge (density) of supermembrane as

$$Z_{\mu\nu} = -\{X_\mu, X_\nu\} \quad (4.14)$$

Note that we obtain no fivebrane charge $Z_{\mu_1 \dots \mu_5}$ in the superalgebra of supermembrane theory, which is consistent with the past light-cone gauge analyses of superalgebra [1, 19].

In Matrix Theory as formulated in [10], one can in principle introduce a longitudinal fivebrane charge by using two “canonical conjugate pairs of matrices” [20]. Because of the absence of a transverse fivebrane charge in the basic superalgebra of matrix theory, the issue of the Lorentz invariance of matrix theory becomes very subtle in the presence of the

fivebrane. On the other hand, the superalgebra in supermembrane theory as evaluated here has a manifestly Lorentz covariant form.

In connection with the above path integral evaluation of superalgebra, we want to comment on the following two issues: The first is if the path integral itself is well-defined after $\theta_R = 0$ gauge fixing. The second is a construction of a supercharge which is conserved, or if not conserved, a supercharge which is conserved up to a BRST exact piece, instead of the charge which is simply broken by the term $\tilde{\xi}_L \theta_R$ as in (4.6).

As is explained in the next section, we have a fermionic constraint $\chi_\alpha(\sigma) \approx 0$ (5.4) after the gauge fixing $\theta_R = 0$, and the Poisson bracket of the constraint $\chi_\alpha(\sigma)$ is given by

$$\{\chi_\alpha(\sigma), \chi_\beta(\tilde{\sigma})\}_P = -2i\delta(\sigma - \tilde{\sigma})\Gamma_{\alpha\beta}^m W_m(\sigma) \quad (4.15)$$

with $W_m = P_m - \{X_m, X^{11}\} + 2i\bar{\theta}_L \Gamma_m \{\theta_L, X^{11}\}$. If one treats this constraint as a second-class constraint, one has to add an extra term

$$\mathcal{L}_{\phi_L} = \bar{\phi}_L \Gamma^m W_m \phi_L \quad (4.16)$$

to the total Lagrangian with a *bosonic* left-handed Majorana spinor ϕ_L , and the path integral is defined by

$$Z = \int d\mu \mathcal{D}\phi_L \exp \left[iS + i \int_W d^3\sigma \bar{\phi}_L \Gamma^m W_m \phi_L \right] \quad (4.17)$$

This ϕ_L is analogous to the Faddeev-Popov ghost.¹ The field ϕ_L is non-propagating in the sense of a 3-dimensional field theory, and it may be neglected if one applies a suitable regularization. However, this factor is important when one examines if the path integral is well-defined. In fact, $(\Gamma^m W_m)^2 \approx -(\partial_0 X^{11})^2 - \frac{1}{2}\{\Pi^m, \Pi^n\}^2$ as is explained in (5.26), and it vanishes for a naive vacuum configuration, which suggests that the path integral could be singular even after the gauge fixing $\theta_R = 0$. This singular behavior is presumably related to the well-known instability of a supermembrane for a naive ground state configuration[2]. One can of course stabilize the membrane by a suitable compactification. The non-trivial central charge generally suggests certain compactification, and in this sense our

¹ The origin of this factor is understood if one remembers that the Faddeev-Popov gauge fixing is to add a constraint (gauge condition) $\chi_2 \approx 0$ to make the first class gauge generator $\chi_1 \approx 0$ effectively a second-class constraint. The Faddeev-Popov factor is then given by

$$\det \begin{bmatrix} \{\chi_1, \chi_1\}_P & \{\chi_1, \chi_2\}_P \\ \{\chi_2, \chi_1\}_P & \{\chi_2, \chi_2\}_P \end{bmatrix}^{1/2} = \det\{\chi_1, \chi_2\}_P$$

In the present case, we have the second class constraints χ_α , and the determinant factor becomes $(\det\{\chi_\alpha, \chi_\beta\}_P)^{-1/2}$, where the minus sign in the exponential arises from the fact that we are dealing with fermionic variables. This determinant factor is exponentiated by a bosonic spinor ϕ_L .

evaluation of the possible central charge is well-defined. Moreover, one can make the factor $\Gamma^m W_m$ invariant even under localized SUSY transformation if one uses a variable $p^m = P^m + i\bar{\theta}_L \Gamma^m \{\theta_L, X^{11}\}$ in a first order formalism, which means that \mathcal{L}_{ϕ_L} in (4.16) does not affect the analysis of superalgebra.

We next comment on the supercharge which is conserved up to a BRST exact term. For this purpose, we rewrite the conservation equation (4.6). The equation of motion for θ_R gives

$$\bar{\xi}_L(\sigma) + \frac{\delta S^{(0)}}{\delta \theta_R(\sigma)} = 0 \quad (4.18)$$

where we separated the total action $S = S^{(0)} + S_g$ into a gauge fixing part S_g and the rest. In the remainder of this section, the derivative stands for the right-derivative. Next we note the κ -symmetry of $S^{(0)}$, which amounts to

$$\int_W d^3\sigma \left[\frac{\delta S^{(0)}}{\delta \theta_R(\sigma)} \delta_\kappa \theta_R(\sigma) + \frac{\delta S^{(0)}}{\delta \theta_L(\sigma)} \delta_\kappa \theta_L(\sigma) + \frac{\delta S^{(0)}}{\delta X^\mu(\sigma)} \delta_\kappa X^\mu(\sigma) + \frac{\delta S^{(0)}}{\delta \tilde{g}^{ab}(\sigma)} \delta_\kappa \tilde{g}^{ab}(\sigma) \right] = 0 \quad (4.19)$$

where we use the transformation law in (1.11). Since S_g does not depend on X^μ and θ_L , the equations of motion give $\frac{\delta S^{(0)}}{\delta \theta_L(\sigma)} = \frac{\delta S^{(0)}}{\delta X^\mu(\sigma)} = 0$. The equation of motion for \tilde{g}^{ab} is given by

$$\frac{\delta S^{(0)}}{\delta \tilde{g}^{ab}(\sigma)} + \frac{\delta S_g}{\delta \tilde{g}^{ab}(\sigma)} = 0 \quad (4.20)$$

If one combines the above 3 equations, one obtains

$$\bar{\xi}_L(\sigma) = -\frac{\delta S_g}{\delta \tilde{g}^{ab}(\sigma)} \bar{\psi}_c^a(\sigma) \tilde{g}^{cb}(\sigma) \quad (4.21)$$

where we defined

$$K_b^a(\sigma) \equiv \bar{\psi}_b^a(\sigma) \kappa_R(\sigma) \quad (4.22)$$

in eq.(1.11) by noting (2.8).

For the specific gauge fixing Lagrangian (2.10), we obtain

$$\begin{aligned} -\bar{\xi}_L &= N_a \bar{\psi}_c^0 \tilde{g}^{ca} - i(\partial_b b_a) c^b \bar{\psi}_c^0 \tilde{g}^{ca} - i b_a \left[\partial_b c^0 \bar{\psi}_c^b \tilde{g}^{ca} + \partial_b c^a \bar{\psi}_c^0 \tilde{g}^{cb} \right] \\ &= N_a \bar{\psi}_c^0 \tilde{g}^{ca} + i b_a \left[\partial_b (c^b \bar{\psi}_c^0 \tilde{g}^{ca}) - \partial_b c^0 \bar{\psi}_c^b \tilde{g}^{ca} - \partial_b c^a \bar{\psi}_c^0 \tilde{g}^{cb} \right] \\ &\quad - i \partial_b (b_a c^b \bar{\psi}_c^0 \tilde{g}^{ca}) \\ &= \delta_{BRST} (b_a \bar{\psi}_c^0 \tilde{g}^{ca}) - i \partial_b (b_a c^b \bar{\psi}_c^0 \tilde{g}^{ca}) \end{aligned} \quad (4.23)$$

In the last equation, we used the fact that the transformation property of $\bar{\psi}_c^a \kappa_R \tilde{g}^{cb}$ under reparametrization symmetry is the same as \tilde{g}^{ab} , since κ -symmetry does not interfere with

the reparametrization symmetry. As κ_R is a scalar quantity under reparametrization symmetry, $\bar{\psi}_c^a \tilde{g}^{cb}$ also has the same transformation property as \tilde{g}^{ab} .

We can thus rewrite eq.(4.6) as

$$\partial_a \tilde{J}^a = i\delta_{BRST} (b_a \psi_c^0 \tilde{g}^{ca}) \quad (4.24)$$

with

$$\tilde{J}^a \equiv J^a - b_b c^a \psi_c^0 \tilde{g}^{cb} \quad (4.25)$$

One can also understand (4.2) up to equations of motion as

$$\delta_\epsilon S = \int_W d^3\sigma \left(i\partial_a \bar{\epsilon} \tilde{J}^a - \bar{\epsilon} \delta_{BRST} (b_a \psi_c^0 \tilde{g}^{ca}) \right) + \int_W \frac{1}{2} d\beta_2 \quad (4.26)$$

By starting with

$$\langle \bar{\eta} \tilde{Q}(\tilde{\tau}) \rangle \quad (4.27)$$

where $\tilde{Q}(\tilde{\tau}) \equiv \int_\Sigma d^2\sigma \tilde{J}^0(\tilde{\tau}, \sigma^k)$, one can derive the Ward-Takahashi identity as before, and one obtains the same algebra as (4.12)

$$[\bar{\epsilon} \tilde{Q}(\tau), \bar{\eta} \tilde{Q}(\tau)] = \int_\Sigma d^2\sigma \bar{\epsilon} \left(2\Gamma^\mu P_\mu(\tau, \sigma^k) - \Gamma^{\mu\nu} \{X_\mu, X_\nu\}(\tau, \sigma^k) \right) \eta \quad (4.28)$$

In deriving this relation, it is important to recognize that $\psi_c^0(\sigma)$ depends on $\theta(\sigma)$ only through its derivative $\partial_a \theta(\sigma)$ or SUSY invariant combination Π_a^μ . This means that the variation of the extra term in \tilde{J}^a under a τ -dependent but σ^k -independent supersymmetry transformation $\theta \rightarrow \theta + \epsilon(\tau)$ always gives rise to terms proportional to $\partial_\tau \epsilon(\tau)$. These terms in turn give rise to terms proportional to $\partial_\tau \delta(\tau - \tilde{\tau})$, which are removed when one goes to the T-product from T*-product by BJL prescription[18]. Consequently, the extra term in \tilde{J}^a does not modify the supercharge algebra.

The supercharge \tilde{Q} defined in terms of \tilde{J}^a is analogous to the conserved supercharge in the light-cone gauge[1, 7] which is obtained by a suitable combination of the localized SUSY transformation and κ -transformation. Since the reparametrization gauge fixing of \tilde{g}^{0a} partly spoils κ -symmetry, a choice of κ_R , which compensates the variation of θ_R under a localized SUSY transformation ϵ_R , does not generate a conserved charge; instead one obtains \tilde{Q} defined above.

5 Superalgebra via Dirac bracket

In this section, we present a Dirac bracket analysis of superalgebra on the basis of the Lagrangian (3.17), which directly leads to the commutator algebra in quantized theory.

The conjugate momenta of X^m , X^{11} and θ_L are calculated from Lagrangian (3.17) as

$$P_m = \Pi_{m0} + i\bar{\theta}_L \Gamma_m \{X^{11}, \theta_L\} \quad (5.1)$$

$$P_{11} = \partial_0 X_{11} + i\bar{\theta}_L \Gamma_m \{\theta_L, X^m\} \quad (5.2)$$

$$\bar{S}_R = -i\bar{\theta}_L \Gamma^m (P_m + Y_m) \quad (5.3)$$

where Y_m is defined in (3.22). From the definition of (5.1) and (5.3), we have a primary constraint

$$\bar{\chi}_R = \bar{S}_R + i\bar{\theta}_L \Gamma^m (P_m + Y_m) \approx 0 \quad (5.4)$$

The constraint $\bar{\chi}_R$ forms a second class constraint (and consequently, there is no secondary constraint). Using the standard Poisson bracket relations

$$\begin{aligned} \{S_{R\alpha}(\sigma), \theta_L^\beta(\tilde{\sigma})\}_P &= \delta_\alpha^\beta \delta(\sigma - \tilde{\sigma}) \\ \{X^m(\sigma), P^n(\tilde{\sigma})\}_P &= \eta^{mn} \delta(\sigma - \tilde{\sigma}) \end{aligned} \quad (5.5)$$

the Poisson bracket of the constraint χ_α is calculated as

$$\begin{aligned} C_{\alpha\beta}(\sigma - \tilde{\sigma}) &\equiv \{\chi_\alpha(\sigma), \chi_\beta(\tilde{\sigma})\}_P \\ &= -2i\delta(\sigma - \tilde{\sigma}) \Gamma_{\alpha\beta}^m W_m(\sigma) \\ (C^{-1})_{\tilde{\beta}}^\alpha(\sigma - \tilde{\sigma}) &= \frac{i}{2} \delta(\sigma - \tilde{\sigma}) \frac{\Gamma^{m\alpha}_{\tilde{\beta}} W_m}{W^2} \end{aligned} \quad (5.6)$$

where we defined

$$W_m = P_m - \{X_m, X^{11}\} + 2i\bar{\theta}_L \Gamma_m \{\theta_L, X^{11}\} = \Pi_{m0} + Y_m \quad (5.7)$$

The Dirac bracket is generally defined as

$$\{f, g\}_D = \{f, g\}_P - \int d^2\sigma d^2\tilde{\sigma} \{f, \chi_\alpha(\sigma)\}_P (C^{-1})_{\tilde{\beta}}^\alpha(\sigma - \tilde{\sigma}) \{\chi^{\tilde{\beta}}(\tilde{\sigma}), g\}_P \quad (5.8)$$

Supersymmetry with a parameter ϵ_L is manifest in the $\theta_R = 0$ gauge. The corresponding supercharge is obtained by the Noether procedure as

$$Q_R = \int_\Sigma d^2\sigma \, 2\Gamma_m \theta_L \left[\Pi_0^m - \{X^m, X^{11}\} + \frac{1}{3} \{\Sigma^m, dX^{11}\} \right] \quad (5.9)$$

For a supersymmetry transformation with a parameter ϵ_R , it is broken by the gauge fixing term. We can nevertheless identify the supercharge Q_L for the broken symmetry by simply

substituting the gauge conditions $\tilde{g}^{0a} + \delta^{0a} = 0$ and $\theta_R = 0$ into the supercharge (4.3) defined from the original Lagrangian

$$Q_L = \int_{\Sigma} d^2\sigma \left(2P^{11}\theta_L - \Gamma^{mn}\theta_L\{X_m, X_n\} + \frac{2}{3}\theta_L\{\Sigma_m, dX^m\} - \frac{2}{3}\Gamma^{mn}\{\Sigma_m, dX_n\} - \frac{1}{5}\Gamma^{mn}\theta_L\{\Sigma_m, \Sigma_n\} \right) \quad (5.10)$$

We can establish that Q_L and Q_R form a superalgebra in terms of the Dirac bracket. Using the data of Poisson brackets, for example,

$$\begin{aligned} \{Q_{\alpha}^R, Q_{\beta}^R\}_P &= -4\Gamma_{\alpha\beta}^m \int_{\Sigma} d^2\sigma \bar{\theta}_L \Gamma_m \{\theta_L, X^{11}\} \\ \{Q_{\alpha}^R, \chi_{\beta}\}_P &= -2\Gamma_{\alpha\beta}^m W_m \\ \{Q_L^{\alpha}, \chi_{\beta}\}_P &= 2\delta_{\beta}^{\alpha} \partial_0 X^{11} - \Gamma_{mn\beta}^{\alpha} \{\Pi^m, \Pi^n\} \end{aligned} \quad (5.11)$$

we can calculate the Dirac bracket of supercharges

$$[\bar{\epsilon}_L Q_R, \bar{\eta}_L Q_R]_D = 2\bar{\epsilon}_L \Gamma^m \eta_L \int_{\Sigma} d^2\sigma (P_m - \{X_m, X_{11}\}) \quad (5.12)$$

$$\begin{aligned} [\bar{\epsilon}_L Q_R, \bar{\eta}_R Q_L]_D &= 2 \int_{\Sigma} d^2\sigma \left(P^{11} \bar{\epsilon}_L \Gamma_{11} \eta_R - \frac{1}{2} \bar{\epsilon}_L \Gamma^{mn} \eta_R \{X_m, X_n\} \right. \\ &\quad \left. + \int_{\Sigma} d \left[i(\tilde{l}^{mn} l_m - \tilde{l}^{11} l^n)(2dX_n + \Sigma_n) + \frac{4i}{3} \tilde{l}^{11} l^n \Sigma_n \right] \right) \end{aligned} \quad (5.13)$$

where we defined $[\ , \]_D \equiv i\{ \ , \ \}_D$, $l^m = i\bar{\epsilon}_L \Gamma^m \theta_L$, $\tilde{l}^{11} = i\bar{\eta}_R \Gamma^{11} \theta_L$ and $\tilde{l}^{mn} = i\bar{\eta}_R \Gamma^{mn} \theta_L$. The last term in (5.13), which depends on θ , is neglected in the following. These algebraic relations are also readily derived by the path integral method by starting with $\langle \bar{\eta}_L Q_R \rangle$ or $\langle \bar{\eta}_R Q_L \rangle$ and using the BJL procedure in Lagrangian (3.17).

We compare these Dirac brackets with the chiral decomposition of (4.13),

$$\begin{aligned} \{Q_{\alpha}^R, Q_{\beta}^R\} &= 2 \left[(P_m + Z_{m11}) \Gamma_{\alpha\beta}^m + \frac{1}{5!} Z_{m_1 \dots m_5} \Gamma_{\alpha\beta}^{m_1 \dots m_5} \right] \\ \{Q_{\alpha}^R, Q_{\beta}^L\} &= 2 \left[P^{11} C_{\alpha\beta} + \frac{1}{2} Z_{mn} \Gamma_{\alpha\beta}^{mn} - \frac{1}{4!} Z_{m_1 \dots m_4 11} \Gamma_{\alpha\beta}^{m_1 \dots m_4} \right] \\ \{Q_{\alpha}^L, Q_{\beta}^L\} &= 2 \left[(P_m - Z_{m11}) \Gamma_{\alpha\beta}^m + \frac{1}{5!} Z_{m_1 \dots m_5} \Gamma_{\alpha\beta}^{m_1 \dots m_5} \right] \end{aligned} \quad (5.14)$$

The central charge density can be read off from (5.12) and (5.13) as ,

$$Z_{m11} = -\{X_m, X_{11}\} \ , \ Z_{mn} = -\{X_m, X_n\} \quad (5.15)$$

It is crucial to observe here that we can reproduce the full algebra from the commutators of $\{Q_{\alpha}^R, Q_{\beta}^R\}$ and $\{Q_{\alpha}^R, Q_{\beta}^L\}$ without using the algebra $\{Q_{\alpha}^L, Q_{\beta}^L\}$ whose evaluation is involved in the present strictly $\theta_R = 0$ gauge, as is explained below.

In the light-cone gauge, the kinetic terms of X^μ and θ are disentangled and they have the standard form[1]. So the Dirac bracket in the light-cone gauge is very simple. In our $SO(9, 1)$ invariant formulation, the Dirac bracket is more involved. Some of the representative Dirac brackets are given by

$$\begin{aligned}\{\theta_L^\alpha(\sigma), \theta_L^\beta(\tilde{\sigma})\}_D &= \frac{i}{2} \frac{W^{\alpha\beta}}{W^2} \delta(\sigma - \tilde{\sigma}) \\ \{X^m(\sigma), X^n(\tilde{\sigma})\}_D &= \frac{i}{2} \frac{\bar{\theta}_L \Gamma^m W \Gamma^n \theta_L}{W^2} \delta(\sigma - \tilde{\sigma}) \\ \{X^m(\sigma), P^n(\tilde{\sigma})\}_D &= \eta^{mn} \delta(\sigma - \tilde{\sigma}) - \frac{i}{2} \frac{\bar{\theta}_L \Gamma^m W \Gamma^n \theta_L}{W^2} \{X^{11}(\sigma), \delta(\sigma - \tilde{\sigma})\}\end{aligned}\quad (5.16)$$

where $W = W_m \Gamma^m$.

For the sake of completeness, we here present a Dirac bracket analysis of the algebra $[\bar{\epsilon}_R Q_L, \bar{\eta}_R Q_L]_D$. For this purpose, it turned out to be simpler to work with the gauge $\theta_R = 0$ but without the gauge fixing of reparametrization symmetry, which avoids an analysis of the ghost sector. We thus start with the Nambu-Goto-type Lagrangian

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{NG} + \mathcal{L}_{WZ} \\ \mathcal{L}_{NG} &= -\sqrt{-h} \\ \mathcal{L}_{WZ} &= i\bar{\theta}_L \Gamma_m [\partial_0 \theta_L \{X^m, X^{11}\} + \partial_0 X^m \{X^{11}, \theta_L\} + \partial_0 X^{11} \{\theta_L, X^m\}] \\ h_{ab} &= \Pi_a^m \Pi_{mb} + \partial_a X^{11} \partial_b X^{11}\end{aligned}\quad (5.17)$$

which is obtained if we integrate out \tilde{g}^{ab} and set $\theta_R = 0$ in (1.1) and (1.4). The conjugate momenta are defined by

$$\begin{aligned}P_m &= p_m + i\bar{\theta}_L \Gamma_m \{X^{11}, \theta_L\} \\ P_{11} &= p_{11} + i\bar{\theta}_L \Gamma_m \{\theta_L, X^m\} \\ \bar{S}_R &= -i\bar{\theta}_L \Gamma^m (P_m + Y_m)\end{aligned}\quad (5.18)$$

where $p_m = -\sqrt{-h}(h^{-1})^{0a} \Pi_{ma}$, $p_{11} = -\sqrt{-h}(h^{-1})^{0a} \partial_a X^{11}$ and $Y_m = -\{\Pi_m, dX^{11}\}$.

There are second-class constraints

$$\bar{\chi}_R = \bar{S}_R + i\bar{\theta}_L \Gamma^m (P_m + Y_m) \approx 0 \quad (5.19)$$

and first-class constraints, which correspond to the generators of reparametrization symmetry,

$$\begin{aligned}B'_0 &= \frac{1}{2} (p_m^2 + p_{11}^2 + \det h_{kl}) \approx 0 \\ B'_k &= p_m \Pi_k^m + p_{11} \partial_k X^{11} \approx 0\end{aligned}\quad (5.20)$$

The Poisson bracket of second-class constraint is given by

$$\{\chi_\alpha(\sigma), \chi_\beta(\tilde{\sigma})\}_P = -2i\delta(\sigma - \tilde{\sigma})\Gamma_{\alpha\beta}^m W_m \quad (5.21)$$

with $W_m = p_m + Y_m$.

The supercharge has the same form as in (5.9) and (5.10), if we replace Π_0^m and $\partial_0 X^{11}$ by p^m and p^{11} , respectively. The Dirac bracket $[\bar{\epsilon}_R Q_L, \bar{\eta}_R Q_L]_D$ is given by (see (5.8) and (5.11))

$$\begin{aligned} [\bar{\epsilon}_R Q_L, \bar{\eta}_R Q_L]_D &= \int_\Sigma d^2\sigma \frac{2}{W^2} \bar{\epsilon}_R (p_{11} - Z) W (-p_{11} - Z) \eta_R \\ &= \int_\Sigma d^2\sigma \frac{2}{W^2} \bar{\epsilon}_R \left\{ (-p_{11}^2 + Z^2) W + [Z, W] p_{11} + \frac{1}{2} [[Z, W], Z] \right\} \eta_R \end{aligned} \quad (5.22)$$

where $Z = \frac{1}{2} \Gamma^{mn} \{\Pi_m, \Pi_n\}$ and $Z^2 = -\frac{1}{2} \{\Pi_m, \Pi_n\}^2$. The commutator of Γ -matrices can be evaluated by noting $[\Gamma^m, \Gamma^{nl}] = 2(\eta^{mn} \Gamma^l - \eta^{ml} \Gamma^n)$ as

$$\begin{aligned} [Z, W] p_{11} + \frac{1}{2} [[Z, W], Z] &= -2W^m \{\Pi_m, \Pi_n\} (p_{11} \eta^{nl} + \{\Pi^n, \Pi^l\}) \Gamma_l \\ &\approx -2(-p_{11} Y_n + Y^m \{\Pi_m, \Pi_n\}) (p_{11} \eta^{nl} + \{\Pi^n, \Pi^l\}) \Gamma_l \\ &= -2(-p_{11}^2 + Z^2) Y_m \Gamma^m \end{aligned} \quad (5.23)$$

We used the relation

$$p^m \{\Pi_m, \Pi_n\} = -p_{11} Y_n + \{\mathbf{B}', \Pi_n\} \approx -p_{11} Y_n \quad (5.24)$$

in the second step in (5.23) by noting the constraints (5.20). $[\bar{\epsilon}_R Q_L, \bar{\eta}_R Q_L]_D$ is then reduced to

$$[\bar{\epsilon}_R Q_L, \bar{\eta}_R Q_L]_D = 2 \int_\Sigma d^2\sigma \frac{-p_{11}^2 + Z^2}{W^2} (W_m - 2Y_m) \bar{\epsilon}_R \Gamma^m \eta_R \quad (5.25)$$

If we use

$$\begin{aligned} W^2 &= -p_{11}^2 + Z^2 + 2B'_0 - 2\{\mathbf{B}', dX^{11}\} \\ &\approx -p_{11}^2 + Z^2 \\ &= -p_{11}^2 - \frac{1}{2} \{\Pi_m, \Pi_n\}^2 \end{aligned} \quad (5.26)$$

by noting (5.20), we obtain the final result

$$\begin{aligned} [\bar{\epsilon}_R Q_L, \bar{\eta}_R Q_L]_D &\approx 2\bar{\epsilon}_R \Gamma^m \eta_R \int_\Sigma d^2\sigma (W_m - 2Y_m) \\ &= 2\bar{\epsilon}_R \Gamma^m \eta_R \int_\Sigma d^2\sigma (P_m + \{X_m, X_{11}\}) \\ &= 2\bar{\epsilon}_R \Gamma^m \eta_R \int_\Sigma d^2\sigma (P_m - Z_{m11}) \end{aligned} \quad (5.27)$$

which has a form expected from (5.12) and (5.14). To make our calculation well-defined, we have to satisfy $-W^2 > 0$ in (5.26). Our analysis of $[\bar{\epsilon}_R Q_L, \bar{\eta}_R Q_L]_D$ is analogous to the Hamiltonian analysis of superalgebra for D-branes in [16].

6 Area preserving symmetry

Our action (3.17) after integrating out the fields \tilde{g}^{ab} , N_a and $\tilde{\xi}$ has a symmetry under a shift of \mathbf{c} by a (fermionic) time-independent Hamiltonian vector field $\vec{\phi}$; $\delta\mathbf{c} = \vec{\phi}$. (This property also holds for the action in (3.1), but we analyze only (3.17) here. Also, one can shift \mathbf{c} by a more general vector field \mathbf{u} with $\text{div}\mathbf{u} = 0$) The generator of this symmetry is given by [12]

$$V = *\mathbf{d}\mathbf{b} = \partial_1 b_2 - \partial_2 b_1 \quad (6.1)$$

The area-preserving diffeomorphism (APD) of a general dynamical variable \mathcal{O} is given by a Lie derivative with respect to a (bosonic) time-independent Hamiltonian vector field \vec{w} ;

$$\delta\mathcal{O} = -\mathcal{L}_{\vec{w}}\mathcal{O} \quad (6.2)$$

For example, $\mathcal{L}_{\vec{w}}X^\mu = \{w, X^\mu\}$. The generator of APD is defined by

$$\begin{aligned} L &= *\mathbf{d}\mathbf{B} \\ &= \{P_m, X^m\} + \{P_{11}, X^{11}\} + \{\bar{S}_R, \theta_L\} \\ &\quad + i\{b_0, c^0\} + i(\mathbf{b}, \vec{c}) + 2iVc + i(\mathbf{d}V, \mathbf{c}) \end{aligned} \quad (6.3)$$

where P_m, P_{11} and \bar{S}_R are defined in (5.1)-(5.3). It is shown that the two generators V and L form a BRST multiplet [21]

$$L = \delta_{BRST}V \quad (6.4)$$

which is physically understood if one remembers that L generates a reparametrization with a parameter of the form \vec{w} .

The symmetries generated by V and L are characterized by time-independent parameters and thus analogous to the residual symmetry in $A_0 = 0$ gauge for Yang-Mills theory. We now discuss how to promote the symmetries generated by V and L to time-dependent gauge symmetry. To gauge the APD symmetry, we rewrite the Lagrangian (in a first order formalism) by introducing new *independent* variables P_m, P_{11} as

$$\begin{aligned} \tilde{\mathcal{L}} &= P_m(\Pi_0^m + i\bar{\theta}_L\Gamma^m\{X^{11}, \theta_L\}) + P_{11}(\partial_0 X^{11} + i\bar{\theta}_L\Gamma^m\{\theta_L, X_m\}) - \frac{1}{2}P_m^2 - \frac{1}{2}P_{11}^2 \\ &\quad + \frac{1}{2}(\bar{\theta}_L\Gamma^m\{X^{11}, \theta_L\})^2 + \frac{1}{2}(\bar{\theta}_L\Gamma^m\{\theta_L, X_m\})^2 - \frac{1}{2}\det G - i\bar{\theta}_L\Gamma^m\partial_0\theta_L Y_m \\ &\quad + ib_0(\partial_0 c^0 - \text{div}\mathbf{c}) + i(\mathbf{b}, \partial_0\mathbf{c}) \end{aligned} \quad (6.5)$$

If we integrate out P_m and P_{11} , we go back to the original Lagrangian (3.17). The Lagrangian (6.5) has an APD symmetry if we assign a transformation property to P_m

and P_{11} as functions on Σ . The APD generator has the same form as (6.3), but P_m and P_{11} are now independent variables.

We now introduce a BRST doublet A, λ [12]

$$\delta_{BRST} A = -i\lambda \quad , \quad \delta_{BRST} \lambda = 0 \quad (6.6)$$

as gauge fields for L and V , respectively. If we add a BRST exact term

$$\mathcal{L}_{exact} = -\delta_{BRST}(AV) = i\lambda V - AL \quad (6.7)$$

to the Lagrangian (6.5), which does not change the physical contents of the theory, and if we integrate out P_m and P_{11} , the Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{gauged} = & \frac{1}{2}(D_0 X^m - i\bar{\theta}_L \Gamma^m D_0 \theta_L)^2 + \frac{1}{2}(D_0 X^{11})^2 - \frac{1}{2} \det G_{kl} \\ & + i\bar{\theta}_L \Gamma_m (D_0 \theta_L \{X^m, X^{11}\} + D_0 X^m \{X^{11}, \theta_L\} + D_0 X^{11} \{\theta_L, X^m\}) \\ & + b_0(D_0 c^0 - \text{div} \mathbf{c}) + i(\mathbf{b}, D_0 \mathbf{c} + \vec{\lambda}) \end{aligned} \quad (6.8)$$

where D_0 is an ‘‘APD covariant derivative’’ defined by

$$D_0 \mathcal{O} = \partial_0 \mathcal{O} + \mathcal{L}_{\vec{A}} \mathcal{O} \quad (6.9)$$

The Lagrangian \mathcal{L}_{gauged} has an area preserving gauge symmetry

$$\begin{aligned} \delta_V \mathbf{c} &= \vec{\phi} \quad , \quad \delta_V \lambda = -D_0 \phi \\ \delta_L \mathcal{O} &= -\mathcal{L}_{\vec{w}} \mathcal{O} \quad , \quad \delta_L A = -D_0 w \end{aligned} \quad (6.10)$$

for *time dependent* functions $\phi(\tau, \sigma^k)$ and $w(\tau, \sigma^k)$. Note that the Lagrangian (6.8) has a structure quite different from that of the M(atric) theory Lagrangian of Refs.[1][10], and it is not given by a simple dimensional-reduction of $D = 10$ super Yang-Mills theory. The Lagrangian (3.17) corresponds to the gauge fixing $A = \lambda = 0$ of this area preserving gauge symmetry.

7 $SO(9, 1)$ covariant matrix regularization

The physical state conditions in our formulation (3.17) are given by

$$\begin{aligned} V|\text{phys}\rangle &= d\mathbf{b}|\text{phys}\rangle = 0 \\ L|\text{phys}\rangle &= d\mathbf{B}|\text{phys}\rangle = 0 \end{aligned} \quad (7.1)$$

,which are the Gauss-law constraints for the $A = \lambda = 0$ gauge in (6.8), in addition to the BRST invariance $Q_{BRST}|\text{phys}\rangle = 0$. We now *locally* solve $V = 0$ at the operator level

$$\mathbf{b} = -\text{d}b \quad (7.2)$$

and treat $c = \text{div} \mathbf{c}$ as an independent variable. This procedure is shown to correspond to a gauge fixing of the symmetry generated by V by a gauge condition $F = \partial_1 c^2 - \partial_2 c^1 = 0$ [12]. The Lagrangian (3.17) is then written as

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}(\Pi_0^m)^2 + \frac{1}{2}(\partial_0 X^{11})^2 - \frac{1}{2} \det G' + ib_0(\partial_0 c^0 - c) + ib\partial_0 c \\ & + i\bar{\theta}_L \Gamma_m \left[\partial_0 \theta_L \{X^m, X^{11}\} + \partial_0 X^m \{X^{11}, \theta_L\} + \partial_0 X^{11} \{\theta_L, X^m\} \right] \end{aligned} \quad (7.3)$$

with

$$\begin{aligned} \det G' = & \frac{1}{2} \{ \Pi^m, \Pi^n \}^2 + \{ \Pi^m, \text{d}X^{11} \}^2 \\ & + 2i \{ \text{d}b, \Pi^m \} \{ \Pi_m, \text{d}c^0 \} + 2i \{ b, X^{11} \} \{ X^{11}, c^0 \} - 3 \{ b, c^0 \}^2 \end{aligned} \quad (7.4)$$

It is crucial that all the terms in (7.3) which contain derivatives with respect to the variables (σ^1, σ^2) are written in terms of the ‘‘Poisson bracket of functions’’ on Σ . We can thus matrix-regularize the Lagrangian in a formal way by the ‘‘correspondence principle’’ [1];

$$\begin{aligned} \mathcal{O}_A(\tau) Y^A(\sigma^1, \sigma^2) & \rightarrow \mathcal{O}_A(\tau) T^A \\ \int_{\Sigma} \text{d}^2 \sigma & \rightarrow \text{Tr} \\ \{ \ , \ } & \rightarrow -i[\ , \] \end{aligned} \quad (7.5)$$

for a generic dynamical variable \mathcal{O} ; $\{Y^A(\sigma^1, \sigma^2)\}$ are a complete set of orthonormal eigenfunctions of Laplacian on Σ , and $\{T^A\}$ are the generators of $SU(N)$ with $N \rightarrow \infty$.

The matrix-regularized action is then written as

$$\begin{aligned} S = & \int \text{d}\tau \text{Tr} \left[\frac{1}{2}(\Pi_0^m)^2 + \frac{1}{2}(\partial_0 X^{11})^2 - \frac{1}{2} \det G' + ib_0(\partial_0 c^0 - c) + ib\partial_0 c \right. \\ & \left. + \bar{\theta}_L \Gamma_m \left(\partial_0 \theta_L [X^m, X^{11}] + \partial_0 X^m [X^{11}, \theta_L] + \partial_0 X^{11} [\theta_L, X^m] \right) \right] \end{aligned} \quad (7.6)$$

where

$$\begin{aligned} \det G' = & -\frac{1}{2} \left([X^m, X^n] - i\bar{\theta}_L \Gamma^m [\theta_L, X^n] - i\bar{\theta}_L \Gamma^n [X^m, \theta_L] + \bar{\theta}_L \Gamma^m [\theta_L, \bar{\theta}_L]_+ \Gamma^n \theta_L \right)^2 \\ & - \left([X^m, X^{11}] - i\bar{\theta}_L \Gamma^m [\theta_L, X^{11}] \right)^2 \\ & - 2i \left([b, X^m] + i\bar{\theta}_L \Gamma^m [b, \theta_L]_+ \right) \left([X_m, c^0] - i\bar{\theta}_L \Gamma_m [\theta_L, c^0]_+ \right) \\ & - 2i [b, X^{11}] [X^{11}, c^0] + 3 [b, c^0]_+^2 \end{aligned} \quad (7.7)$$

The bracket $[b, c^0]_+$, for example, stands for an anti-commutator of matrix valued fermionic variables.

The Hamiltonian and the generator of the area preserving diffeomorphism are respectively represented in matrix formulation as

$$H = \text{Tr} \left(\frac{1}{2} p^\mu p_\mu + \frac{1}{2} \det G' + i b_0 c \right) \quad (7.8)$$

and

$$\text{Tr}(wL') = \text{Tr} w \left(-i[P_m, X^m] - i[P_{11}, X^{11}] - i[\bar{S}_R, \theta_L]_+ + [b_0, c^0]_+ + [b, c]_+ \right) \quad (7.9)$$

where the parameter $w(\sigma^1, \sigma^2)$ is also represented by an infinite dimensional matrix. In the Hamiltonian above we defined the variables

$$\begin{aligned} p_m &\equiv P_m - \bar{\theta}_L \Gamma_m [X^{11}, \theta_L] \\ p_{11} &\equiv P_{11} - \bar{\theta}_L \Gamma_m [\theta_L, X^m] \\ \bar{S}_R &= -i\bar{\theta}_L \Gamma^m (P_m + i[X_m, X^{11}] + \bar{\theta}_L \Gamma_m [\theta_L, X^{11}]) \end{aligned} \quad (7.10)$$

In the Hamiltonian formulation we have a (second class) constraint

$$\chi = \bar{S}_R + i\bar{\theta}_L \Gamma^m (P_m + i[X_m, X^{11}] + \bar{\theta}_L \Gamma_m [\theta_L, X^{11}]) \approx 0 \quad (7.11)$$

which complicates practical manipulations, although the Hamiltonian itself has a relatively simple form as in (7.8).

The supercharge is defined in the chiral decomposition as

$$Q_R = \text{Tr} \left[2\Gamma_m \theta_L \left(P^m - \bar{\theta}_L \Gamma^m [X^{11}, \theta_L] + i[X^m, X^{11}] - \frac{1}{3} [X^{11}, \bar{\theta}_L] \Gamma^m \theta_L \right) \right] \quad (7.12)$$

which corresponds to the Noether charge for $\delta\theta_L = \epsilon_L$ and $\delta X^\mu = i\bar{\epsilon}_L \Gamma^\mu \theta_L$, and

$$\begin{aligned} Q_L = \text{Tr} & \left(2P^{11} \theta_L + i\Gamma^{mn} \theta_L [X_m, X_n] - \frac{2}{3} \theta_L \bar{\theta}_L \Gamma^m [\theta_L, X_m] \right. \\ & \left. + \frac{2}{3} \Gamma^{mn} [X_n, \bar{\theta}_L] \Gamma_m \theta_L + \frac{i}{5} \Gamma^{mn} \theta_L \bar{\theta}_L \Gamma_m [\theta_L, \bar{\theta}_L] + \Gamma_n \theta_L \right) \end{aligned} \quad (7.13)$$

which corresponds to $\delta\theta_R = \epsilon_R$ and $\delta X^\mu = i\bar{\epsilon}_R \Gamma^\mu \theta_R$ in (3.1); the variable θ_R is set to 0 after the evaluation of the Noether charges.

We here note that the reparametrization BRST charge Q_{BRST} in (3.28) itself does not have a simple matrix representation. For example, the replacement of c^k by a single variable $c = \text{div} \mathbf{c}$ does not go through, and the two variables c^1 and c^2 remain in the

BRST charge till the end. A true significance of this property is not clear, but it is partly related to the fact that we solved $V = 0$ in the operator level but $L = \delta_{BRST}V = 0$ is not solved in the operator level: Instead we impose a constraint $L|\text{phys}\rangle = 0$ on the physical state vector. The manifest BRST invariant formulation in (7.1) is thus partly spoiled by solving $V = 0$ in the above procedure.

The BRST charge Q_{BRST} generates transformation with 3 independent ghost variables (c^0, c^1, c^2) , whereas the matrix formulation above contains only two independent ghost variables $(c^0, c = \text{div}\mathbf{c})$ and corresponding canonical conjugate variables (b_0, b) . This reduction of the number of freedom is related to the symmetries generated by V and L , which exist even after the BRST invariant reparametrization gauge fixing in (2.10). This suggests that a proper use of the area preserving diffeomorphism reduces the BRST invariant physical states to those specified by two independent ghosts $(c^0, c = \text{div}\mathbf{c})$ only. Although we cannot implement this statement in the operator level, we expect that this procedure works in the physical matrix elements formed by BRST invariant physical states.

8 Discussion

We have presented an $SO(9, 1)$ invariant formulation of the 11-dimensional supermembrane by combining an $SO(10, 1)$ invariant treatment of reparametrization symmetry with an $SO(9, 1)$ invariant $\theta_R = 0$ gauge of κ -symmetry. The light-cone gauge formulation[1] and the present $SO(9, 1)$ invariant formulation, for example eq.(6.8), have quite different appearance. We however emphasize that these two formulations in fact describe an *identical* theory (i.e., 11-dimensional supermembrane) in the common domain where both gauge conditions are well-defined.

Our $SO(9, 1)$ formulation of supermembrane compared to the light-cone gauge formulation preserves a large subset of $D = 11$ Lorentz boost symmetry, which may be regarded as “dynamical” symmetry. However, the rotational symmetry between “M-direction” (X^{11} -direction) and the other directions is not manifest. Recently $D = 11$ Lorentz symmetry has been checked in a $D2$ -brane scattering with M-momentum transfer [22] and in the analysis of Lorentz algebra in the light-cone gauge [19]. A similar check need to be done in our formalism as to the Lorentz-algebra and also the explicit calculation of dynamical processes.

It is known that Green-Schwarz type IIA string action is obtained by a “double dimensional reduction” of the supermembrane action[4]. The $\theta_R = 0$ gauge becomes singular

for a naive “double dimensional reduction” due to the denominator $\hat{\gamma}^2$ in γ_{ir} (2.6), and in this sense our $SO(9,1)$ invariant formulation may be regarded as intrinsic to the supermembrane. However, $\theta_R = 0$ gauge is smoothly related to the configuration which has no winding in M-direction, i.e., the $D2$ -brane.

Recently the supermembrane with non-trivial winding has been studied [23, 24]. In these analyses, harmonic 1-forms on Σ play an essential role. In our treatment of V and L symmetries, we considered only Hamiltonian vector fields. In order to treat the non-trivial topology of Σ and the supermembrane with winding, we have to consider area-preserving diffeomorphism (APD) corresponding to a “locally Hamiltonian vector field”, which is a symplectic dual of the harmonic 1-form. We have additional constraints associated with “locally Hamiltonian vector fields” \mathbf{u}_i ($i = 1, \dots, 2g$) on Σ of genus g

$$V_i = \int_{\Sigma} d^2\sigma(\mathbf{b}, \mathbf{u}_i) \approx 0 \quad (8.1)$$

$$L_i = \int_{\Sigma} d^2\sigma(\mathbf{B}, \mathbf{u}_i) = \delta_{BRST} V_i \approx 0 \quad (8.2)$$

which are the generators of symmetries $\delta_{V_i} \mathbf{c} = \epsilon \mathbf{u}_i$ and $\delta_{L_i} \mathcal{O} = -\mathcal{L}_{\mathbf{u}_i} \mathcal{O}$, respectively. When we solved the constraint $d\mathbf{b} = 0$ in (7.2), we neglected the harmonic part of \mathbf{b} , which is justified only when $\Sigma = S^2$. It is generally necessary to consider the effect of harmonic 1-forms when we matrix-regularize the supermembrane with genus $g \geq 1$.

As to the relevance of our formulation to the so-called M-theory, a better understanding of the fundamental degrees of freedom in M-theory is important. The M(atrix) theory in [10] is formulated by regarding $D0$ -branes as fundamental degrees of freedom. A crucial observation is the decoupling of anti- $D0$ -branes in the infinite momentum frame, and it allows them to treat only $D0$ -branes as fundamental degrees of freedom. But in a general Lorentz frame, the interaction between $D0$ -branes and anti- $D0$ -branes cannot be ignored in general, and a deeper understanding of the fundamental degrees of freedom is required. The basic idea of “membrane as composites of $D0$ -branes” by Townsend[25], which is one of the physical bases of M(atrix) theory, is based on the resemblance of the light-cone gauge action of supermembrane (APD SYM_{0+1})[1] and the effective action of N coincident $D0$ -branes ($U(N)$ SYM_{0+1}) [26]. Our $SO(9,1)$ invariant Lagrangian with APD gauge symmetry (6.8) does not correspond to a dimensional reduction of $D = 10$ supersymmetric Yang-Mills theory, and a direct relation to $D0$ -branes is lost. However, our Lagrangian and light-cone Lagrangian describe the identical physics as was emphasized above, and we hope that our formulation may shed new light on the basic dynamics of $D0$ -branes and anti- $D0$ -branes, and possibly on the dynamics of M-theory itself.

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